(Exact) Lifted inference

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Work with: http://minervaintelligence.com, https://treatment.com/

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1. Representations: Problog & MLNs
2. Representation Issues
3. **(Exact) Lifted Inference**
4. Lifted Approximate Inference and Optimization
5. Learning
6. Applications

(Exact) Lifted Inference

Outline

1. (Exact) Lifted Inference
   - Recursive Conditioning
   - Lifted Recursive Conditioning

Why Exact Inference?

Why do we care about exact inference?

- Gold standard

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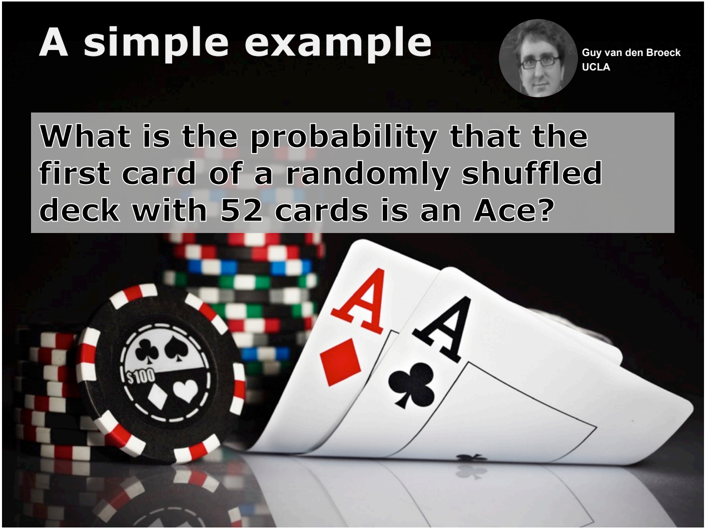
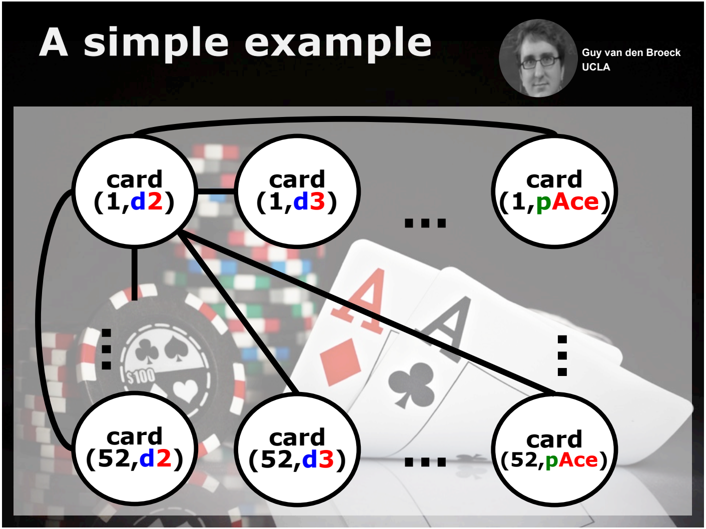
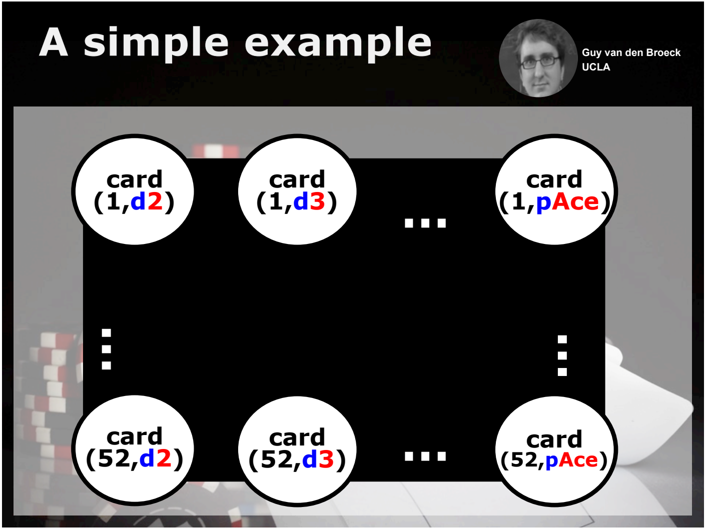
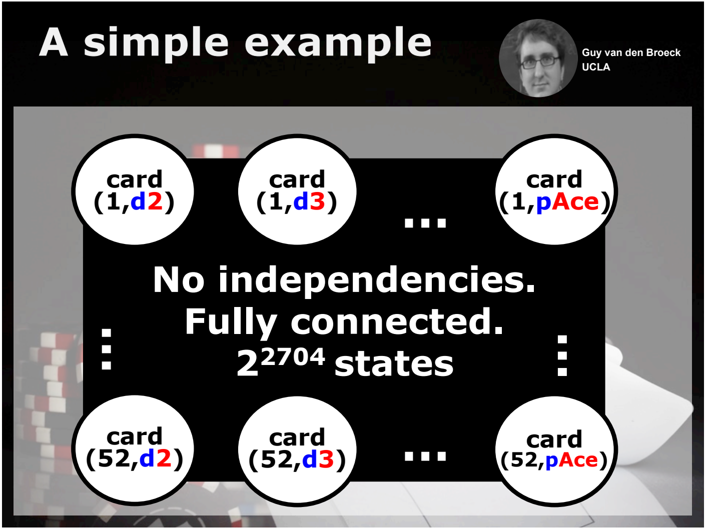
Why do we care about exact inference?

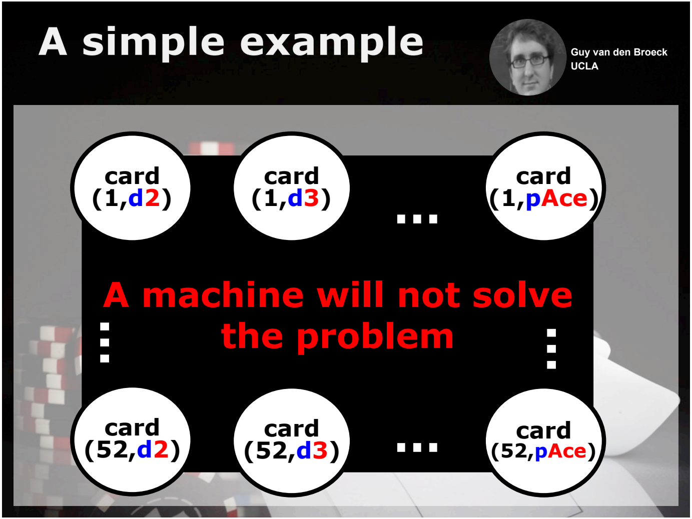
- Gold standard
- Size of problems amenable to exact inference is growing
- Learning for inference
- Basis for efficient approximate inference:
  - Rao-Blackwellization
  - Variational Methods

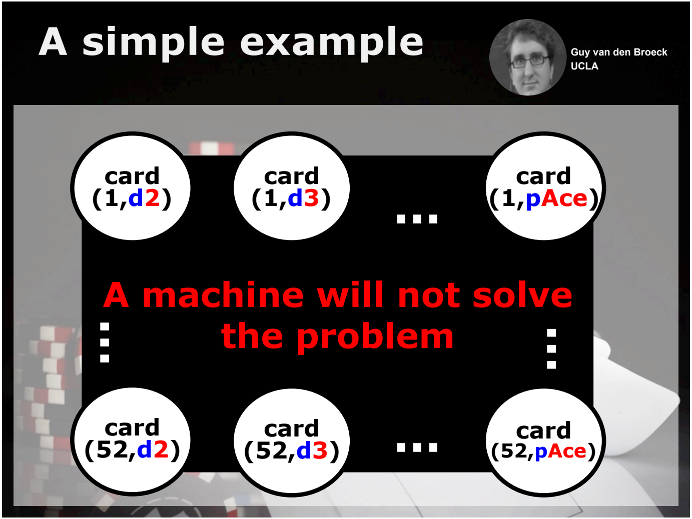

A simple example
Guy van den Broeck
UCLA
What is the probability that the first card of a randomly shuffled deck with 52 cards is an Ace?
$100


A simple example
Guy van den Broeck
UCLA
card
(1,d2)
card
(1,d3)
...
card
(1,pAce)
card
(52,d2)
card
(52,d3)
...
card
(52,pAce)


A simple example
Guy van den Broeck
UCLA
card (1,d2)
card (1,d3)
...
card (1,pAce)
card (52,d2)
card (52,d3)
...
card (52,pAce)


A simple example
Guy van den Broeck
UCLA
card (1,d2)
card (1,d3)
...
card (1,pAce)
No independencies.
Fully connected.
2^2704 states
card (52,d2)
card (52,d3)
...
card (52,pAce)


A simple example
Guy van den Broeck
UCLA
card (1,d2)
card (1,d3)
...
card (1,pAce)
A machine will not solve the problem
card (52,d2)
card (52,d3)
...
card (52,pAce)



. . . unless it can represent and exploit symmetry.

Example: lifted inference

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- We don't need to reason about all of the other individuals separately, but can count over them.

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- A reasonable model about the eloquence of the speaker might depend on the questions asked
- each of the people who didn't ask a question, their silence might depend on the questions asked, but not on the questions not asked.
- Rather than reasoning separately about each person who was observed to not ask a question, it is reasonable to just count over them.

Example: lifted inference

- The spread of a malaria (or other diseases) may depend on the number of people and the number of mosquitoes.
- Individual mosquitoes are important in such a model, but we don't want to model each mosquito separately.

Lifted Inference

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- Use the ideas from lifted theorem proving - no need to ground.
- Potential to be exponentially faster in the number of non-differentialed individuals.
- Relies on knowing the number of individuals (the population size).

Outline

1. (Exact) Lifted Inference
   - Recursive Conditioning
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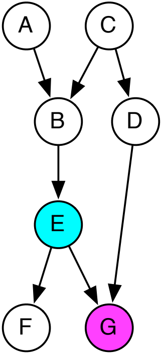
Inference via factorization in graphical models

$$P(E \mid g) = \frac{P(E \wedge g)}{\sum_E P(E \wedge g)}$$



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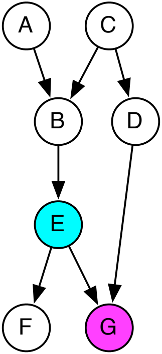

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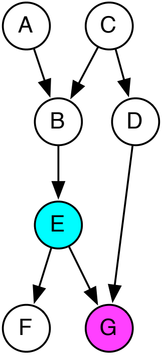
$$P(E \wedge g)$$

$$= \sum_F \sum_B \sum_C \sum_A \sum_D P(A)P(B \mid AC) P(C)P(D \mid C)P(E \mid B)P(F \mid E)P(g \mid ED)$$

$$=$$

Inference via factorization in graphical models



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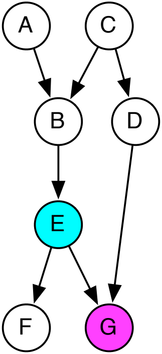

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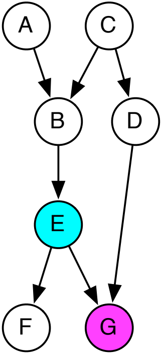

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$$\sum_C \left(P(C) \left(\sum_A P(A)P(B \mid AC) \right) \left(\sum_D P(D \mid C)P(g \mid ED) \right) \right)$$

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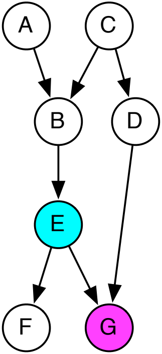

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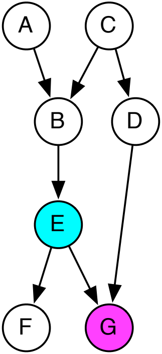
Inference via factorization in graphical models



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$$= \left(\sum_F P(F \mid E)\right) \sum_B P(E \mid B) \sum_C \left(P(C) \left(\sum_A P(A)P(B \mid AC)\right) \left(\sum_D P(D \mid C)P(g \mid ED)\right)\right)$$

Recursive Conditioning

- Computes sum (partition function) from outside in

Input:

- Context - assignment of values to variables
- Set of factors

Output: value of summing out other variables (partition function)

- Evaluate a factor as soon as all its variables are assigned
- Cache values already computed
- Recognize disconnected components
- Recursively branch on a variable

Recursive Conditioning

procedure $rc(Con : \text{context}, \quad Fs : \text{set of factors})$:
- if $\exists v$ such that $\langle\langle Con, Fs\rangle, v\rangle \in cache$
  - return v
- else if $vars(Con) \not\subseteq vars(Fs)$
  - return $rc(\{X = v \in Con : X \in vars(Fs)\}, Fs)$
- else if $\exists F \in Fs$ such that $vars(F) \subseteq vars(Con)$
  - return $eval(F, Con) \times rc(Con, Fs \setminus \{F\})$
- else if $Fs = Fs_1 \uplus Fs_2$ where $vars(Fs_1) \cap vars(Fs_2) \subseteq vars(Con)$
  - return $rc(Con, Fs_1) \times rc(Con, Fs_2)$
- else select variable $X \in vars(Fs)$
  - $sum \leftarrow 0$
  - for each $v \in domain(X)$
    - $sum \leftarrow sum + rc(Con \cup \{X = v\}, Fs)$
  - $cache \leftarrow cache \cup \{\langle\langle Con, Fs\rangle, sum\rangle\}$
  - return sum

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  else if $\exists F \in Fs$ such that $vars(F) \subseteq vars(Con)$
    return $eval(F, Con) \times rc(Con, Fs \setminus \{F\})$ Evaluate
  else if $Fs = Fs_1 \uplus Fs_2$ where $vars(Fs_1) \cap vars(Fs_2) \subseteq vars(Con)$
    return $rc(Con, Fs_1) \times rc(Con, Fs_2)$
  else select variable $X \in vars(Fs)$ Branch
    $sum \leftarrow 0$
    for each $v \in domain(X)$
      $sum \leftarrow sum + rc(Con \cup \{X = v\}, Fs)$
    $cache \leftarrow cache \cup \{\langle\langle Con, Fs\rangle, sum\rangle\}$
    return sum

Recursive Conditioning

procedure $rc(Con : \text{context}, \quad Fs : \text{set of factors})$:

- if $\exists v$ such that $\langle\langle Con, Fs\rangle, v\rangle \in cache$ — Recall
  - return v
- else if $vars(Con) \not\subseteq vars(Fs)$ — Forget
  - return $rc(\{X = v \in Con : X \in vars(Fs)\}, Fs)$
- else if $\exists F \in Fs$ such that $vars(F) \subseteq vars(Con)$
  - return $eval(F, Con) \times rc(Con, Fs \setminus \{F\})$
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  - $cache \leftarrow cache \cup \{\langle\langle Con, Fs\rangle, sum\rangle\}$ — Remember
  - return sum

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  else if $Fs = Fs_1 \uplus Fs_2$ where $vars(Fs_1) \cap vars(Fs_2) \subseteq vars(Con)$
    return $rc(Con, Fs_1) \times rc(Con, Fs_2)$ Disconnected
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    for each $v \in domain(X)$
      $sum \leftarrow sum + rc(Con \cup \{X = v\}, Fs)$
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    return sum

Variable Elimination and Recursive Conditioning

- Variable elimination is the dynamic programming variant of recursive conditioning.
- Recursive Conditioning is the search variant of variable elimination
- They do the same additions and multiplications.
- Complexity $O(nr^t)$, for n variables, range size r, and treewidth t.

Outline

Weighted Formula

A Weighted formula is a pair $\langle F, v\rangle$ where

- F a formula on parametrized random variables
- v number

Example:

$\langle X \neq Y \land likes(X,Y) \land rich(Y), 0.001\rangle$
$\langle likes(X,X) \land rich(X), 0.7\rangle$

...

Lifted Recursive Conditioning

LiftedRC(Context, WeightedFormulas)

- *Context* is a set of assignments to random variables and counts to assignments of instances of relations. e.g.:

$$\begin{aligned}\{\neg a,\ & \#_X f(X) \land g(X) = 7, \\ & \#_X f(X) \land \neg g(X) = 5, \\ & \#_X \neg f(X) \land g(X) = 18, \\ & \#_X \neg f(X) \land \neg g(X) = 0\}\end{aligned}$$

- *WeightedFormulas* is a set of weighted formulae, e.g.,

$$\begin{aligned}\{ & \langle \neg a \land \neg f(X) \land g(X), 0.1 \rangle, \\ & \langle a \land \neg f(X) \land g(X), 0.2 \rangle, \\ & \langle f(X) \land g(Y), 0.3 \rangle, \\ & \langle f(X) \land h(X), 0.4 \rangle \}\end{aligned}$$

Evaluating Weighted Formulae

Context:

$$\{\neg a, \quad \#_X f(X) \land g(X) = 7,$$
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WeightedFormulas:

$$\{ \quad \langle \neg a \land \neg f(X) \land g(X), 0.1 \rangle,$$
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$LiftedRC(Context, WeightedFormulas)$ returns:

Evaluating Weighted Formulae

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LiftedRC(*Context*, *WeightedFormulas*) returns:

$$0.1^{18} * 1 *$$

Evaluating Weighted Formulae

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WeightedFormulas:

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LiftedRC(*Context*, *WeightedFormulas*) returns:

$$0.1^{18} * 1 * 0.3^{12*}$$

Evaluating Weighted Formulae

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LiftedRC(*Context*, *WeightedFormulas*) returns:

$$0.1^{18} * 1 * 0.3^{12*25} *$$

Evaluating Weighted Formulae

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WeightedFormulas:

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$LiftedRC(Context, WeightedFormulas)$ returns:

$$0.1^{18} * 1 * 0.3^{12*25} * LiftedRC(Context, \{\langle f(X) \wedge h(X), 0.4 \rangle\})$$

Branching

Context:

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WeightedFormulas: $\{\langle f(X) \land h(X), 0.4\rangle, \ldots\}$
Branching on H for the 7 "X" individuals s.th. $f(X) \land g(X)$:
$LiftedRC(Context, WeightedFormulas) =$

Branching

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$$\sum_{i=0}^{7} \binom{7}{i} LiftedRC(\{\neg a,\ \#_X f(X) \wedge g(X) \wedge h(X) = i, \#_X f(X) \wedge g(X) \wedge \neg h(X) = 7 - i, \#_X f(X) \wedge \neg g(X) = 5, \dots\}, WeightedFormulas)$$

Branching

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$$WeightedFormulas)$$

Branching

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$$\#_X \neg f(X) \wedge g(X) = 18,$$
$$\#_X \neg f(X) \wedge \neg g(X) = 0\}$$

WeightedFormulas: $\{\langle f(X) \wedge h(X), 0.4\rangle, \dots\}$
Branching on H for the 7 "X" individuals s.th. $f(X) \wedge g(X)$:
$LiftedRC(Context, WeightedFormulas) =$

$$\sum_{i=0}^{7} \binom{7}{i} LiftedRC(\{\neg a,\ \#_X f(X) \wedge g(X) \wedge h(X) = i,$$
$$\#_X f(X) \wedge g(X) \wedge \neg h(X) = 7 - i,$$
$$\#_X f(X) \wedge \neg g(X) = 5, \dots\},$$
$$WeightedFormulas)$$

Branching

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$$\{\neg a,\ \#_X f(X) \wedge g(X) = 7,\ \#_X f(X) \wedge \neg g(X) = 5,\ \#_X \neg f(X) \wedge g(X) = 18,\ \#_X \neg f(X) \wedge \neg g(X) = 0\}$$

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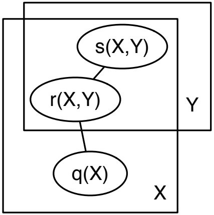
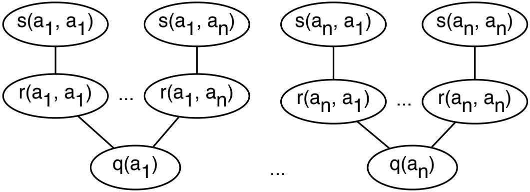
Recognizing Disconnectedness



Relational Model



Grounding

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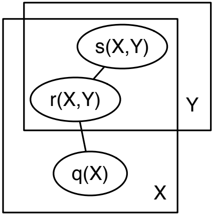

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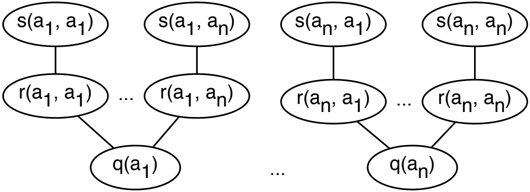

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$$LiftedRC(Context, WeightedFormulas)$$
$$= LiftedRC(Context, WeightedFormulas\{X/c\})^n$$

...now we only have unary predicates

Observations and Queries

- Observations become the initial context.
  Observations can be ground or lifted.
-
  $$P(q|obs) = \frac{LiftedRC(q \wedge obs, WFs)}{LiftedRC(q \wedge obs, WFs) + LiftedRC(\neg q \wedge obs, WFs)}$$

  calls can share the cache
- "How many?" queries are also allowed

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As the population size n of undifferentiated individuals increases:

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Always exponentially faster than grounding everything.

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We can lift a model that consists just of

$$\langle \{f(X) \land g(Z)\}, \alpha_4 \rangle$$

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We can lift a model that consists just of

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Compilation

- The computation reduces to products and sums
- The structure can be determined at compile time
- Orders of magnitude faster than lifted recursive conditioning
- Often abstracted as weighted model counting (WMC)

Take Home

- Lifted inference exploits symmetries ("for all")
- Instead of considering which individuals a predicate is true for, count how many individuals it is true for, and determine appropriate probabilities.
- Always exponentially better in the number of undifferentiated individuals than grounding everything.
- Open problem: finding a dichotomy of those problems we know we can lift and those we know it is impossible to lift.

What is now required is to give the greatest possible development to mathematical logic, to allow to the full the importance of relations, and then to found upon this secure basis a new philosophical logic, which may hope to borrow some of the exactitude and certainty of its mathematical foundation. If this can be successfully accomplished, there is every reason to hope that the near future will be as great an epoch in pure philosophy as the immediate past has been in the principles of mathematics. Great triumphs inspire great hopes; and pure thought may achieve, within our generation, such results as will place our time, in this respect, on a level with the greatest age of Greece.

– Bertrand Russell 1917